THE NATIONAL TECHNICAL UNIVERSITY OF UKRAINE
"Igor Sikorsky Kyiv Polytechnic Institute"

Emblem departments (if available)

Department of Mathematical Physics and Differential Equations

NAME OF THE COURSE

Higher mathematics. Part 3. Field theory. Rows. Functions of a Complex Variable

Working program of the academic discipline (Syllabus)

Details of the academic discipline

Level of higher education	***First (bachelor)***
Branch of knowledge	*17 Electronics, automation and electronic communications*
Specialty	*174 Automation, computer-integrated technologies and robotics*
Educational program	*Technical and software automation tools*
Status of Discipline	*Normative*
Form of education	*Full-time*
Year of training, semester	*Second year, autumn semester*
Scope of the discipline	*150 / 5 credits*

	Lectures	Practical classes (seminars)	Laboratory classes (computer practice)	Individual classes	Self-study work of student
Hours	**36**	**36**	**0**	**0**	**78**

Semester control/ control measures	Exam	Test	Modular control work *(specify quantity)*	Calculation graphic work, calculation work *(specify quantity)*	Home control work *(specify quantity)*	Abstract *(specify quantity)*
	+	-	1	0	1	0

Lessons schedule	*On the website of the university, as well as the website of the Faculty of Chemical Engineering*
Language of teaching	*English*
Information about head of the course / teachers	Lecturer: Lystopadova Valentyna, Ph.D., associate professor of the Department of Mathematical Physics and Differential Equations, Faculty of Physics and Mathematics listopadova17@ukr.net http://intellect.kpi.ua/profile/lvv61 ORCID: https://orcid.org/0000-0002-2549-8381 Practical: Lystopadova Valentyna, Ph.D., associate professor of the Department of Mathematical Physics and Differential Equations, Faculty of Physics and Mathematics listopadova17@ukr.net http://intellect.kpi.ua/profile/lvv61 ORCID: https://orcid.org/0000-0002-2549-8381
Placement of the course	Website of the department, information resources in the library

Program of educational discipline

Description of the educational discipline, its purpose, subject of study and learning outcomes

The goal of the educational discipline is the formation of students' integral competence — the ability to think logically, the formation of students' personalities; development of their intelligence and abilities; the ability to solve complex specialized tasks and practical problems, characterized by complexity and uncertainty of conditions, during professional activities in the field of automation, computer-integrated technologies and robotics or in the learning process; use methods of mathematical analysis in engineering calculations.

Software competencies:

General competences (GC)
GC1 Ability to think abstractly.
GC2 Ability to apply knowledge of mathematics, to the extent necessary for the use of mathematical methods for the analysis and synthesis of automation systems.
GC3 Ability to apply methods of system analysis, mathematical modeling, identification and numerical methods to develop mathematical models of individual elements and automation systems as a whole, to analyze the quality of their functioning using the latest computer technologies.

Program learning outcomes (LO)

LO1 Know linear and vector algebra, differential and integral calculus, functions of several variables, functional series, differential equations for functions of one and many variables, operational calculus, theory of functions of a complex variable, theory of probabilities and mathematical statistics, theory of random processes to the extent necessary for using of mathematical apparatus and methods in the field of automation.

LO2 To be able to apply methods of system analysis, modeling, identification and numerical methods to develop mathematical and simulation models of individual elements and automation systems as a whole, to analyze the quality of their functioning using the latest computer technologies.

Pre-requisites and post-requisites of the discipline (place in the structural and logical scheme of training according to the relevant educational program)

Higher mathematics-3 is taught in the third semester on the basis of full secondary or secondary professional education. In the structural and logical scheme of the training program in this direction, the academic discipline "Higher Mathematics" precedes and provides the following academic disciplines: Mathematical methods in automation tasks, Theory of automatic control, Computer modeling of processes and systems.

Content of the academic discipline

1. *Integral calculus of a function of several variables. Elements of field theory*. Double integral. Triple integral. Curvilinear integrals of the first and second kind. Surface integrals of the first and second kind. Elements of vector analysis and field theory.
2. *Rows.* Number series. Functional series. Power series. Fourier series.
3. *Theory of functions of a complex variable.* Derivative and differential of a function of a complex variable. Integration of functions of a complex variable. Row of Laurent. Leftovers.

Educational materials and resources

Basic literature

1. Дубовик В.П. Вища математика: навч. посіб. / В.П. Дубовик, І.І. Юрик. – К.: Ігнатекс-Україна, 2013. – 648 с.

2. Дубовик В.П. Вища математика: Збірник задач: навч. посіб. / В.П. Дубовик, І.І. Юрик, І.П. Вовкодав та ін. – К.: Ігнатекс-Україна, 2011. – 480с.

3. Грималюк В.П. Вища математика: У 2 ч.: навч. посіб. / Грималюк В.П., Кухарчук М.М., Ясінський В.В. – К.: Віпол, 2004. – Ч. 2. – 400 с.

4. Герасимчук В. С. Вища математика. Повний курс у прикладах і задачах: навч. посіб.: Кратні, криволінійні та поверхневі інтеграли. Елементи теорії поля. Ряди. Прикладні задачі/ В.С. Герасимчук, Г. С. Васильченко, В. І. Кравцов. – Київ : Книги України ЛТД, 2009. – 400 с.

5. Клепко В. Ю. Вища математика в прикладах і задачах: навчальний посібник / В.Ю.Клепко, В.Л. Голець. – К.: Центр навчальної літератури, 2017. – 594 с.

6. Зайцев Є. П. Вища математика: інтегральне числення функцій однієї та багатьох змінних, звичайні диференціальні рівняння, ряди: навч. посіб. / Є. П. Зайцев. – К.: Алерта, 2018. – 608 с.

7. Вища математика. Числові та функціональні ряди [Електронний ресурс]: навчальний посібник для здобувачів ступеня бакалавра за природничимиі технічними спеціальностями/ КПІ ім. Ігоря Сікорського; уклад.: О.В.Борисенко, В.В.Листопадова.- Електронні текстові дані (1файл: 1.78 Мбайт).- Київ: КПІ ім. Ігоря Сікорського, 2023.- 117с. Доступ: https://ela.kpi.ua/handle/123456789/67184

8.Авдєєва Т.В. Кратні інтеграли. Елементи теорії поля: Методичні вказівки та завдання до виконання індивідуальної роботи для студентів інженерних спеціальностей/ Т.В. Авдєєва, О.Б. Качаєнко, О.О. Коваль, О.Б. Поліщук, В.І. Стогній. – К.: ІВЦ "Видавництво «Політехніка»", 2016,– 92 с. Доступ: http://kmf.kpi.ua/

Additional literature

1. Шкіль М.І., Колесник Т.В. Вища математика. - К.: Вища школа, 1986. – 512 с.
2. Стрижак Т.Г. Математичний аналіз: приклади і задачі: навч. посіб. / Стрижак Т.Г., Коновалова Н.Р. – К.: Либідь, 1995. – 240 с.
3. Авдєєва Т.В., Качаєнко О.Б. Ряди Фур'є. Практикум. – К.: НТУУ «КПІ», 2016. – 88 с. Доступ: http://ela.kpi.ua/handle/123456789/16839
4. Журавська, Г. В. Теорія функції комплексної змінної [Електронний ресурс] : навчальний посібник для інженерних спеціальностей / Г. В. Журавська ; КПІ ім. Ігоря Сікорського. – Електронні текстові дані (1 файл: 2,26 Мбайт). – Київ : КПІ ім. Ігоря Сікорського, 2017 − 92 с. Доступ: https://ela.kpi.ua/handle/123456789/19900

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Educational content

Methods of mastering an educational discipline (educational component)

5.1. Didactic materials:

In lecture classes – Lecture (electronic version), explanation, brainstorming, problem tasks

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List of lectures

Lecture 1. Definition of double integral. Properties, its calculation

1.1. The concept of a double integral.
1.2. Conditions of existence and properties.
1.3. Geometric and physical content.
1.4. Calculation of the double integral.

Lecture 2. Change of variables in the double integral. Application of double integrals

2.1. Change of variables in the double integral. Jacobian concept.
2.2. Double integral in polar coordinates.
2.3. Application of double integrals in geometry.
2.4. Application of double integrals in mechanics.

Lecture 3. Triple integral

3.1. The concept of the triple integral.
3.2. Conditions of existence and properties.
3.3. Geometric and physical content.
3.4. Calculation of the triple integral.

Lecture 4. Change of variables in the triple integral. Application of triple integrals

4.1. Change of variables in the triple integral.
4.2. Triple integral in cylindrical and spherical coordinates.
4.3. Application of triple integrals in geometry.
4.4. Application of triple integrals in mechanics.

Lecture 5. Curvilinear integrals of the first kind (along the length of the arc). Curvilinear integrals of the second kind (by coordinates)

5.1. The concept of a curvilinear integral of the first kind.
5.2. Calculation of the curve integral of the first kind.
5.3. Application of the curvilinear integral of the first kind. .
5.4. The concept of a curvilinear integral of the second kind. Physical content.
5.5. Calculation of the curvilinear integral of the second kind.
5.6. Application of the curvilinear integral of the second kind.

Lecture 6. Green's formula. Conditions for the independence of the curvilinear integral from the form of the path of integration. . Scalar and vector fields, their classification and characteristics

6.1. Connection between the curve integral of the first and second kind.
6.2. Green's formula.
6.3. Conditions for the independence of the curvilinear integral from the form of the path of integration.
6.4. Scalar field, its characteristics.
6.4.1. Derivative in direction.
6.4.2. Gradient.
6.5. Vector field, its characteristics.
6.5.1. Divergence.
6.5.2. Potential field. Vector field rotor.

Lecture 7. Surface integrals of the first kind (by surface area). Surface integrals of the second kind (by coordinates)

7.1. Definition of surface integrals of the first kind. Conditions of existence.
7.2. Calculation of surface integrals of the first kind.
7.3. Definition of surface integrals of the second kind. Conditions of existence.
7.4. Calculation of surface integrals of the second kind.
7.5. Field flow.

Lecture 8. The Ostrogradsky–Gauss formula. Divergence. Stokes formula

8.1. The Ostrogradsky–Gauss formula.
8.2. Stokes formula.

Lecture 9. Numerical series

9.1. Basic concepts and definitions.
9.2. Geometric progression. Harmonic series.
9.3. Properties of convergent numerical series.
9.4. Necessary condition for convergence of the series.

Lecture 10. Comparing series with Positive Terms. Sufficient signs of convergence

10.1. Comparison theorems.
10.2. Dalembert's sign.
10.3. Cauchy's radical sign.
10.4. Cauchy's integral sign.

Lecture 11. Alternating series. Absolute and conditional convergence

11.1. The concept of a sign-changing series.
11.2. Leibniz's theorem.
11.3. Absolute and conditional convergence.

Lecture 12. Functional series. Power series

12.1. The interval of convergence of the functional series.
12.2. Major series. Uniform convergence.
12.3. Properties of functional series.
12.4. Power series. Abel's theorem.
12.5. The interval and radius of convergence of the power series.
12.6. Properties of power series.

Lecture 13. Taylor series. Development of basic elementary functions into Maclauren series. Application of power series to approximate calculations

13.1. Taylor and McLaren series.

13.2. Breakdown into a number of basic elementary functions.

13.3. Binomial series.

13.4. Application of power series to approximate calculations

13.4.1. Approximate solution of differential equations.

13.4.2. Approximate calculations of function values.

13.4.3. Approximate calculation of definite integrals.

Lecture 14. Trigonometric Fourier series

14.1. Trigonometric Fourier series. Fourier coefficients.

14.2. Fourier series for even and odd functions.

14.3. Fourier series for 2l-periodic functions.

Lecture 15. The concept of a function of a complex variable. . Differentiation of a function of a complex variable

15.1. Definition of the function of a complex variable. Single-valued and multi-valued functions.

15.2. Real and imaginary parts of a function of a complex variable.

15.3. Continuity of a function of a complex variable.

15.4. Basic elementary functions.

15.5. A necessary and sufficient condition for differentiability of a function of a complex variable. Differential of a function of a complex variable. Cauchy-Riemann conditions.

15.6. Analytical function and its properties.

Lecture 16. Integration of functions of a complex variable

16.1. The original Calculating the integral of an analytic function in the case of a known primitive.

16.2. Cauchy's integral formula.

16.3. The Cauchy integral.

16.4. Derivatives of higher orders from an analytic function.

Lecture 17. Power series of functions of a complex variable. Row of Laurent.

17.1. Taylor series.

17.2. Row of Laurent.

17.3. Special points of a unique analytic function.

17.4. Expansion of a function in the Laurent series around a regular point; around the pole; in the vicinity of a significant special point.

Lecture 18. Leftovers. Calculation of integrals of a function of a complex variable and definite improper integrals of a real variable. Logarithmic excess

18.1. The concept of an integral remainder of an analytic function.

18.2. Calculation of integral residues at isolated singular points of single-valued functions.

18.3. Calculation of integrals of a function of a complex variable and definite improper integrals of a real variable. Lema Jordan.

18.4. Logarithmic derivative. Logarithmic excess.

18.5. Calculation of logarithmic remainders.

In practical classes – typical tasks and tasks of increased complexity, which make it possible to learn basic mathematical concepts and their properties, to develop the ability to use the material for independent problem solving.

List of (approximate) practical classes

Practical lesson 1. Calculation of double integrals.

Practical lesson 2. Change of variables (polar coordinate system) in double integrals. Application of double integrals.

Practical lesson 3. Calculation of triple integrals.

Practical lesson 4. Cylindrical and spherical coordinate systems. Application of triple integrals.

Practical lesson 5. Calculation of curvilinear integrals of the 1st kind for different forms of specifying a plane curve and a curve in three-dimensional space. Application of curvilinear integrals of the first kind.

Practical lesson 6. Calculation of curvilinear integrals of the II kind for different forms of specifying a flat curve and a curve in three-dimensional space. Application of curvilinear integrals of the II kind. Conditions for the independence of the curvilinear integral from the path of integration on the plane. Conditions for the independence of the curvilinear integral from the path of integration in three-dimensional space. Green's formula.

Practical lesson 7. Surface integrals of the first kind.

Practical lesson 8. Surface integrals of the second kind. Field flow.

Practical lesson 9. Elements of field theory. Gradient. Divergence. The vector form of writing the Ostrogradsky formula. Vector circulation. Rotor. Stokes formula. Modular short-term control work -1.

Practical lesson 10. Numerical series with positive terms. Necessary and sufficient signs of convergence of numerical series.

Practical lesson 11. Interchangeable rows. Absolute and conditional convergence of series.

Practical lesson 12. Power series. Abel's theorem. The interval and radius of convergence of the power series.

Practical lesson 13. Taylor and McLaren series. Application of power series to approximate calculations.

Practical lesson 14. Trigonometric Fourier series. Fourier series for even and odd, 2l-periodic functions. Modular short-term control work -2.

Practical lesson 15. Elementary functions of a complex variable. Derivative and differential of a function of a complex variable. Cauchy–Riemann conditions. Reconstruction of an analytic function by its real or imaginary part.

Practical lesson 16. Calculation of the integral of the function of the complex variable along the curve of the complex plane. Calculating the integral of an analytic function in the case of a known primitive. Cauchy's integral theorem. Cauchy's integral formula. An analogue of Cauchy's integral formula for derivatives.

Practical lesson 17. Row of Laurent. Isolated special points of a unique character. Convergence set of the Laurent series. Expansion of a function into a Laurent series.

Practical lesson 18. Calculation of the integral remainder of the analytical function. Calculation of improper integrals. Logarithmic excess. Modular short-term control work -3.

In practical classes - Tasks to be completed (according to the specified list of basic literature).

5.2. Technical support: Microsoft Office Word, any software for the execution of graphic material (at the request of the student).

Self-study work of student

Types of Self-study work – study of lecture material, preparation for classroom classes, problem solving, homework and homework control work (divided into two parts according to the semester plan certifications).

Policies and control

7. Policies of academic discipline (educational component)

Compliance with the provisions of the "Code of Honor of the National Technical University of Ukraine «Igor Sikorsky Kyiv Polytechnic Institute» (chapters 2 and 3). Read more: https://kpi.ua/code. Student cooperation in solving problem tasks is allowed, but each student defends the answers independently. Student interaction during the exam is strictly prohibited and any such activity will be considered a breach of academic integrity in accordance with the university's principles of academic integrity.

8. Types of control and rating system for evaluating learning outcomes (RS)

Current control*:* express survey, survey on the subject of the lesson, writing MCW and HCW.

Calendar control*:* is held twice a semester as a monitoring of the current state of fulfillment of the syllabus requirements.

Semester control: exam.

Terms of admission to semester control: a minimum positive grade for MCW, credit for homework, semester rating of at least 36 points.

The student's rating in the discipline for the semester consists of the points he receives for:

1) Eight answers (per student on average) in practical classes (provided that 7 students are interviewed in one lesson with a maximum group size of 25 people;
2) one modular control work;
3) one homework test;
4) the answer to the exam.

Work in practical classes

Under the condition of good preparation and active work in the practical session, the student receives 1 point. One or two best students in each practical session can be given 1 point as an incentive. The maximum number of points for all practical classes is equal to 1 point x 8 = 8 points.

Final control

One modular control work (MCW) is divided into three parts:

Modular short-term control work -1 "Multiple integrals. Field theory": weighted score – 20 points;

Modular short-term control work -2 "Numerical and functional series": weighted score – 10 points;

Modular short-term control work -3 "Theory of functions of a complex variable": weighted score – 10 points.

(It is allowed to divide the Modular short-term control work into several thematic control papers that have the same total weighting point).

The maximum number of points for all test papers is equal to 40 points.

Home control work

Weight score – 12 points. The work is evaluated in terms of the percentage of correctly solved tasks.

Penalty and incentive points for:

- untimely (later than a week) submission of homework control work ……. – 2 points (for each week late);
- failure to perform homework and independent work …… – 1 point (for each task);
- prizes in faculty and institute Olympiads in higher mathematics; preparation and defense of reports at student scientific and practical conferences, performance of tasks to improve didactic materials from the credit module …….. + 6 points.

Calculation of the rating scale (R):

The sum of the weighted points of control measures during the semester is:

$$R_S = 40 + 20 = 60 \text{ points.}$$

The examination component of the scale is equal to 40% of R, namely:

$$R_E = 40 \text{ points.}$$

Thus, the rating scale for the discipline is:

$$R = R_S + R_E = 100 \text{ points.}$$

The size of the rating scale R=100 points.

Size of the starting scale R_S = 60 points.

The size of the examination scale R_E =40 points.

At the decision of the examiner, without an additional survey, it is possible to assign (with the student's consent) a grade of "good" ("B" or "C" in the ECTS system) in the event that the student's starting rating is at least 0.9 of the maximum possible (R_S), that is, at $R_S \geq 54$ points.

Table of correspondence of rating points to grades on the university scale:

PERSONAL SCORE	*GRADE*
100-95	Excellent
94-85	Very Good
84-75	Good
74-65	Satisfactory
64-60	Sufficiently
Personal score<60	Unsatisfactory
Personal score < 36 or the individual tasks is not credited	Not allowed to pass exam

9. Additional information on the discipline (educational component)

- *the possibility of enrolling in certificates of completion of distance or online courses on the relevant subject.*

Working program of the academic discipline (syllabus):

Compiled
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Approved: Department of Mathematical Physics and Differential Equations (Protocol №9 dated June 26, 2024)
Agreed: Methodical Commission of chemical engineering faculty (Protocol №11 dated June 28, 2024)